Inconsistency management based on relevance degrees

Anna Zamansky

Information Systems Department
University of Haifa
Joint work with Ofer Arieli and Kostas Stefanidis
J.-Y. Béziau, "The future of paraconsistent logic" (1990):

"...Non-monotonists reject monotonicity because they think there are experiences (mostly involving birds) which show monotonicity is wrong. But one who thinks the paraconsistent way would reject the principle of non contradiction, and not monotonicity. His strategy is more imaginative: he accepts to see penguins flying in the sky of Hawaii’s beaches.”
Sources of Inconsistency

- **Databases:**
  - integration of multiple data sources
  - deactivated (or no) integrity enforcement
- **Software development:** contradictory descriptions (models, specifications, designs, program code, user guides, test plans, change requests, style guides, process models...)
- **Collaborative environments**
(In)Consistency Management: Earlier Approach

<table>
<thead>
<tr>
<th>Gärdenfors, 1988</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inconsistency is an <strong>“epistemic hell”</strong> to be avoided by all costs.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gabbay and Hunter, 1991</th>
</tr>
</thead>
<tbody>
<tr>
<td>The consensus of opinion in the logic community is that inconsistency is <strong>undesirable</strong>.</td>
</tr>
</tbody>
</table>
Paraconsistent Trends

- **Databases**: consistent query answering (Arenas, Bertossi and Chomicki), inconsistency management policies (Subrahmanian et al.)
- **Software engineering**: paraconsistent requirement modeling languages (Borgida et al., Nuseibeh)
- **Logic programming**: paracoherent answer set programming (Sakama et al., Eiter et al.)
- **Description logics**: paraconsistent DL formalisms (Ma et al.)
- **Information systems**: inconsistency robustness (Hewitt).
The decision to repair an inconsistency is risk-based. If the cost of fixing it outweighs the risk of ignoring it, then it makes no sense to fix it.

Inconsistency management is preserving inconsistency where it is desirable to do so, identifying inconsistency at the point where decisions are required, remedying inconsistency prior to taking action.
Talk Plan

- Brief Introduction to Paraconsistent logics
- The framework of distance-based semantics
- Context-awareness by relevance degrees.
A logic

1. A formal language $\mathcal{L}$.
2. A consequence relation $\vdash$ for $\mathcal{L}$.

**consequence relation (cr) for $\mathcal{L}$**

A binary relation between sets of $\mathcal{L}$-formulas and $\mathcal{L}$-formulas with:

- **strong reflexivity:** if $\psi \in \Gamma$ then $\Gamma \vdash \psi$.
- **monotonicity:** if $\Gamma \vdash \psi$ and $\Gamma \subseteq \Gamma'$, then $\Gamma' \vdash \psi$.
- **transitivity (cut):** if $\Gamma \vdash \psi$ and $\Gamma, \psi \vdash \varphi$ then $\Gamma \vdash \varphi$. 
How are logics characterized?

- **Semantically:**  \( \Gamma \vdash^S \psi \) if every “model” of \( \Gamma \) is a “model” of \( \psi \) in the semantics \( S \).

- **Syntactically:**  \( \Gamma \vdash^D \psi \) if \( \psi \) has a **proof** from \( \Gamma \) in the deduction system \( D \).
Semantic way of defining CL

- A classical valuation $\nu$ is a model of an $\mathcal{L}$-formula $\psi$ if $\nu(\psi) = \texttt{t}$. $\nu$ is a model of a theory $\Gamma$ if $\nu$ is a model of every $\psi \in \Gamma$.
- $\Gamma \vdash_{CPL} \psi$ if every classical model of $\Gamma$ is a model of $\psi$. 
Syntactic way of defining CL

- Axiom schemata:

  I1 \( \varphi \supset (\psi \supset \varphi) \)
  I2 \( (\varphi \supset \psi \supset \theta) \supset (\varphi \supset \psi) \supset (\varphi \supset \theta) \)
  I3 \( ((\psi \supset \varphi) \supset \psi) \supset \psi \)
  C1 \( \varphi \land \psi \supset \varphi \)
  C2 \( \varphi \land \psi \supset \psi \)
  C3 \( \varphi \supset (\psi \supset \varphi \land \psi) \)
  D1 \( \varphi \supset \varphi \lor \psi \)
  D2 \( \psi \supset \varphi \lor \psi \)
  D3 \( (\varphi \supset \theta) \supset (\psi \supset \theta) \supset (\varphi \lor \psi \supset \theta) \)
  N1 \( \neg \varphi \lor \varphi \)
  N2 \( (\varphi \land \neg \varphi) \supset \psi \)

- Inference Rule:

  \[ \psi \quad \psi \supset \varphi \quad \therefore \varphi \quad \text{MP} \]
The Problem of Trivialization

Problem: classical logic is trivialized in the presence of an inconsistent theory.

Classical Entailment

\[ \Gamma \vdash_{CPL} \psi \iff \text{Models}(\Gamma) \subseteq \text{Models}(\psi) \]
The Problem of Trivialization

Problem: classical logic is trivialized in the presence of an inconsistent theory.

Classical Entailment

\[ \Gamma \vdash_{CPL} \psi \iff \text{Models}(\Gamma) \subseteq \text{Models}(\psi) \]

Solutions:

1. Change your logic \(\Rightarrow\) Paraconsistent logics
2. Handle the inconsistency \(\Rightarrow\) Database repair, Belief revision, Information integration,...
The Problem of Trivialization

Problem: classical logic is trivialized in the presence of an inconsistent theory.

Classical Entailment

\[ \Gamma \vdash_{CPL} \psi \iff \text{Models}(\Gamma) \subseteq \text{Models}(\psi) \]

Solutions:

1. Change your logic \( \Rightarrow \) Paraconsistent logics
2. Handle the inconsistency \( \Rightarrow \) Database repair, Belief revision, Information integration,...
What kind of logic is needed for reasoning with inconsistencies?

Paraconsistent logic - a logic which allows contradictory but non-trivial theories.

**Definition**

A propositional logic $\mathbf{L} = \langle \mathcal{L}, \vdash \rangle$ is *paraconsistent* (with respect to $\neg$) if there are $\mathcal{L}$-formulas $A, B$, such that $A, \neg A \not\vdash B$. 
The fathers of paraconsistent logic

S. Jaśkowski, 1948:
...PL should be rich enough to enable practical inferences.

N.C.A. da Costa, 1963:
...PL should contain as much as possible of classical logic.
A valuation $v$ is a model of an $L$-formula $\psi$ if $v(\psi) \in \{t, \top\}$. $v$ is a model of a theory $\Gamma$ if $v$ is a model of every $\psi \in \Gamma$.

$\Gamma \vdash_K \psi$ if every model of $\Gamma$ is a model of $\psi$.

Paraconsistency: $\psi, \neg \psi \not\vdash_K \varphi$
Example 2: The Brazilian School of Paraconsistent Logics

- Divide propositions into two sorts: consistent and inconsistent ones.
- Reflect this classification within the language.
- In the class of C-systems this is done by employing a special (either primitive or defined) connective $\circ$, where the intuitive meaning of $\circ \varphi$ is “$\varphi$ is consistent”.
- Restrict the explosive character of contradictions: they should be explosive only for consistent formulas:

$$A, \neg A \vdash B \quad \Rightarrow \quad A, \neg A, \circ A \vdash B$$
The Basic Paraconsistent Logic $\textbf{BK}$

The system $\textbf{BK}$ extends the positive fragment of classical logic with the following axioms:

(t) $\neg A \lor A$

(b) $\circ A \rightarrow ((A \land \neg A) \rightarrow B)$

(k) $\circ A \lor (A \land \neg A)$
Let \( \text{ADC} \) be the following set of axioms for \( \# \in \{\land, \lor, \supset\} \):

\[
\begin{align*}
(c) & \quad \neg\neg A \supset A \\
(i_1) & \quad \neg \circ A \supset A \\
o_1 & \quad \circ A \supset \circ (A \# B) \\
(a_\#) & \quad \circ A \land \circ B \supset \circ (A \# B) \\
(e) & \quad A \supset \neg\neg A \\
i_2 & \quad \neg \circ A \supset \neg A \\
o_2 & \quad \circ B \supset \circ (A \# B) \\
a_- & \quad \circ A \supset \circ \neg A
\end{align*}
\]

For \( A \subseteq \text{ADC} \), the system BK\( [A] \) is obtained from BK by adding the axioms in \( A \).
For many years the theory of C-systems lacked two important ingredients:

- 1. Intuitive and useful semantics
- 2. Analytic proof systems (analytic calculi for some particular logics were provided in Carielli et al. 1992, Béziau 1993, Neto and Finger 2007, Gentilini 2011).
Recent Developments

For many years the theory of C-systems lacked two important ingredients:

- 1. Intuitive and useful semantics

Some recent contributions to remedy the problem:

- A systematic method for providing non-deterministic semantics and analytic calculi for practically all major C-systems (Avron, Konikowska and AZ 2012, 2013)
- A fully automatic tool for providing semantics and analytic proof systems for all major C-systems which have finite-valued semantics (Ciabattoni, Lahav, Spendier and AZ 2013).
TINC - Paralyzer

Input Syntax

Examples

\*1(\*1 \ a) \rightarrow a, \ (*2 \ a \ & \ *2 \ b) \rightarrow \ *2(a \rightarrow b), \ *2 \ a \rightarrow \ *2(a \lor b), \ *1(a \& b) \rightarrow (*1 \ a \lor *1 \ b), \ldots

Note that you can compute several axioms at once by concatenating them with ";"!

LK+ and BK

Below you can choose whether to start with LK+ or BK and then submit the axiom(s).

- Start with LK+ (no predefined rules for *1 and *2).
- Start with BK:

\[
\begin{align*}
(\Rightarrow \ *1) & \quad \Gamma, \psi \Rightarrow \Delta \\
& \quad \Gamma \Rightarrow \ *1 \psi, \Delta
\\
(\Rightarrow \ *2) & \quad \Gamma \Rightarrow \psi, \Delta \\
& \quad \Gamma, *2 \psi \Rightarrow \Delta
\\
(\Rightarrow \ *2) & \quad \Gamma, \psi, *1 \psi \Rightarrow \Delta \\
& \quad \Gamma \Rightarrow *2 \psi, \Delta
\end{align*}
\]

Input Axiom:  \((\*2 \ a \ & \ *2 \ b) \rightarrow \ *2(a \rightarrow b)\)
Logics of the future?

Decker, 2005

“...we propose to overcome classical logic foundations by adopting paraconsistency as a foundational concept for future information systems engineering...”

Béziau, 1999, ”The future of paraconsistent logic”

“...Perhaps we can say that paraconsistent logic is the logic of the future and may stay so forever.”
Logics of the future?

Decker, 2005

“...we propose to overcome classical logic foundations by adopting paraconsistency as a foundational concept for future information systems engineering...”

Béziau, 1999, ”The future of paraconsistent logic”

“...Perhaps we can say that paraconsistent logic is the logic of the future and may stay so forever.”

- Lack of efficient automated reasoning tools
- Lack of experimental results
- Strong bias towards classical logic
Combining Approaches

So far completely different approaches:

- Paraconsistent logic
- Manipulating inconsistency

Can we develop a logical framework that can combine them?
Suggested Features

- **Faithful to CL**: coinciding with classical logic for consistent theories.
- **Pragmatic**: allowing not only for living with inconsistency, but also doing something about it.
- **Implementable**: encoded in terms of ‘off-the-shelf’ tools.
- **Flexible**: allowing for various management policies.
The key: distance minimization

**Observation 1**

Pragmatic methods for inconsistency management are based on minimization of a "**distance**" between an “object” and its “description”.

- Databases: “closest” repairs
- Software engineering: “maximally consistent” software requirements
- Answer set programming: “maximally coherent” semi-stable models
A Variant of Preferential Semantics

- **Preferential semantics** (Shoham 1987, Kraus, Lehmann and Magidor 1990): for a theory \( \Gamma \), define a preference order \( \leq_{\Gamma} \) on the space of valuations and draw conclusions according to the \( \leq_{\Gamma} \)-minimal valuations.

- **Distance-based semantics**: a variant of preferential semantics in which valuations “closer” to models of \( \Gamma \) are preferred.
Distance-based Entailments

Classical Entailment

\[ \Gamma \models \psi \iff \text{Models}(\Gamma) \subseteq \text{Models}(\psi) \]
Distance-based Entailments

Classical Entailment

\[ \Gamma \models \psi \iff \text{Models}(\Gamma) \subseteq \text{Models}(\psi) \]

Distance-Based Entailment

\[ \Gamma \models \psi \iff \text{MostPlausibleModels}(\Gamma) \subseteq \text{Models}(\psi) \]

where \( \text{MostPlausibleModels}(\Gamma) \) is the set of valuations which are “closest” to being models of \( \Gamma \) according to a chosen notion of “distance”.
Framework Ingredient 1

Distance on $U$

A total function $d : U \times U \rightarrow \mathbb{R}^+$, s.t.:

- $d(l_1, l_2) = d(l_2, l_1)$
- $d(l_1, l_2) = 0$ iff $l_1 = l_2$

Usually triangular inequality is also added:
$d(l_1, l_3) \leq d(l_1, l_2) + d(l_2, l_3)$. 
An aggregation function $f$

- $f$ is a function from multisets of real numbers to real numbers
- $f$ is non-decreasing
- $f(\{x_1, \ldots, x_n\}) = 0$ iff $x_1 = x_2 = \ldots x_n = 0$, and
- $f(\{x\}) = x$, for every $x \in \mathbb{R}$.

Hereditary functions: $f(\{x_1, \ldots, x_n\}) < f(\{y_1, \ldots, y_n\})$ implies also $f(\{x_1, \ldots, x_n, z_1, \ldots, z_m\}) < f(\{y_1, \ldots, y_n, z_1, \ldots, z_m\})$.

Examples: max, $\Sigma$, avg,...
Γ = \{\psi_1, \ldots, \psi_n\}

\(l\) - a (classical) valuation
\(d\) - a distance
\(f\) - an aggregation function.

\[
d_{d,f}(l, \psi_i) = \min \{d(l, l') \mid l' \models \psi_i\}
\]

\[
\delta_{d,f}(l, \Gamma) = f(\{d_{d,f}(l, \psi_1), \ldots, d_{d,f}(l, \psi_n)\})
\]

\[
\Delta_{d,f}(\Gamma) = \{l \mid \neg \exists l'. \delta_{d,f}(l', \Gamma) < \delta_{d,f}(l, \Gamma)\}
\]
Distance-based Entailments (Arieli 2006)

Distance-based Entailment

\[ \Gamma \vdash_{\Delta_d,f} \psi \iff \Delta_d,f(\Gamma) \subseteq \text{Models}(\psi) \]

- \( \vdash_{\Delta_d,f} \) is faithful to CL w.r.t. consistent theories:
  \[ \text{Models}(\Gamma) = \Delta_d,f(\Gamma) \] for any consistent \( \Gamma \)

- \( \vdash_{\Delta_d,f} \) is paraconsistent.
Distance-based Entailments are not strictly CRs

For any hereditary aggregation function \( f \), \( \vdash \Delta_{d,f} \) is a cautious CR:

- **cautious reflexivity:** if \( \Gamma \) is consistent and \( \psi \in \Gamma \), then \( \Gamma \vdash \Delta_{d,f} \psi \).
- **cautious monotonicity:** if \( \Gamma \vdash \Delta_{d,f} \psi \) and \( \Gamma \vdash \Delta_{d,f} \varphi \), then \( \Gamma, \psi \vdash \Delta_{d,f} \varphi \).
- **cautious transitivity:** if \( \Gamma \vdash \Delta_{d,f} \psi \) and \( \Gamma, \psi \vdash \Delta_{d,f} \varphi \), then \( \Gamma \vdash \Delta_{d,f} \varphi \).
Distance-based Logical Framework

- **Faithful to CL**: coinciding with classical logic for consistent theories.
- **Pragmatic**: Many known database repair, information integration and belief revision operators can be expressed within the framework.
- **Implementable**: distance-based operations can be encoded in terms of ‘off-the-shelf’ tools (ASP, SAT, QBF solvers) (Arieli, Denecker and Bruynooghe 2007).
Example: databases

- A database $\mathcal{DB} = \langle \mathcal{D}, \mathcal{IC} \rangle$:
  - $\mathcal{D}$ - a finite set of atoms
  - $\mathcal{IC}$ - a finite set of formulas
- The meaning of $\mathcal{DB}$:
  \[
  \Gamma_{\mathcal{DB}} = \mathcal{D} \cup \text{CWA}(\mathcal{D}) \cup \mathcal{IC}
  \]
  \[
  \text{CWA}(\mathcal{D}) = \{ \neg p \mid p \notin \mathcal{D} \}
  \]
- $\mathcal{DB}$ is consistent if $\Gamma_{\mathcal{DB}}$ is (classically) satisfiable.
Inconsistent databases and repairs

Distance setting: $DS = \langle d, f \rangle$

$$\Delta_{DS}(DB) = \{ I \in \text{mod}(IC) \mid I' \in \text{mod}(IC) \implies \delta_{DS}(I, D \cup \text{CWA}(D)) \leq \delta_{DS}(I', D \cup \text{CWA}(D)) \}$$

**Correspondence to repairs**

$\mathcal{R}$ is a DS-repair of $DB$ induced by $I \in \Delta_{DS}(DB)$ if

$\mathcal{R} = \{ p \in \text{Atoms}(L) \mid l(p) = T \}$. 
Let $DB = \langle D, IC \rangle$ be a database and DS a distance setting. The following conditions are equivalent:

1. $DB$ is consistent,
2. $\Delta_{DS}(DB) = \text{mod}(\Gamma_{DB})$,
3. $\text{Repairs}_{DS}(DB) = \{D\}$. 

Faithfulness to CL
Is the idea extendable to other logics?

- **Goal**: given a logic $L$, produce $L'$, an inconsistency-tolerant variant of $L$, which coincides with $L$ for consistent theories.
- **Denotational semantics** $S = \langle V, \models \rangle$: a set of valuations $V$ + a satisfaction relation $\models$ between valuations and formulas.
- **Induced logic** $L_S$: $\Gamma \vdash_{L_S} \phi$ if for every $v \in V$, $v \models \psi$ for all $\psi \in \Gamma$ implies $v \models \phi$.
- **Examples**: CL, many-valued logics, non-deterministic logics, Kripke semantics.
Is the idea extendable to other logics?

The framework can be extended to any kind of logics based on denotational semantics (Arieli & AZ 2012):

- separation between the underlying logic and the notion of distance minimization
- generalization of the notion of distances
- exploring useful cases (many-valued, non-deterministic, Kripke semantics)
Context: any information that can be used to characterize the situation of an entity (relevant person, place or object).

Context-awareness: the use of context to provide task-relevant information and services to a user [Abowd, 1999].
Context-Awareness Notion from Information Systems

- **Context**: any information that can be used to characterize the situation of an entity (relevant person, place or object).
- **Context-awareness**: the use of context to provide task-relevant information and services to a user [Abowd, 1999].
- **Question**: Can we make distance minimization sensitive to context?
Example: Database Repairs

<table>
<thead>
<tr>
<th>Name</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
<td>10000</td>
</tr>
<tr>
<td>Bob</td>
<td>100000</td>
</tr>
<tr>
<td>Alice</td>
<td>50000</td>
</tr>
</tbody>
</table>

Integrity constraint: Name → Salary

Observation 2

Measurement of minimization of a “distance” can vary for different contexts.
Idea: use context for personalizing user queries.

Context is modeled by an assignment of values to a finite set of context variables.

\[
\begin{align*}
\text{Weather} &= \text{rainy} \\
\text{Distance} &= 30\text{km}
\end{align*}
\]

Example: Mary prefers restaurants with Russian cuisine on rainy days.

\[
\begin{align*}
\text{Restaurant}(\text{Carlo’s Pizza}) & \quad \text{Restaurant}(\text{Russian Samovar}) \\
\text{preference}(\text{Mary}, \text{Carlo’s Pizza}, \text{rainy}) &= 0.2 \\
\text{preference}(\text{Mary}, \text{Russian Samovar}, \text{rainy}) &= 0.9
\end{align*}
\]
New Ingredients

- A context $C$: variables $\langle c_1, \ldots, c_n \rangle$.
  - A context state: an assignment $S$ such that $S(c_i) \in \text{Range}(c_i)$.
  - States($C$): the set of context states.

- A relevance ranking for a set $\Gamma$ of formulas and a context $C$, is a total function $R : \Gamma \times \text{States}(C) \rightarrow (0, 1]$.

- A context setting: $\text{CS}(\Gamma) = \langle C, S, R \rangle$, where $C$ is a context, $S \in \text{States}(C)$ is a $C$-state, and $R$ is a relevance ranking function.
Let $CS(\mathcal{L}) = \langle C, S, R \rangle$ be a context setting for a language $\mathcal{L}$. A distance setting $DS = \langle d, f \rangle$ is **CS-sensitive**, if for every two atomic formulas $p_1$ and $p_2$ such that $R(p_1, S) > R(p_2, S)$:

$$d_{DS}(l_2, p_1) > d_{DS}(l_1, p_2)$$

for every $l_1 \in \text{mod}(p_1) \setminus \text{mod}(p_2)$ and $l_2 \in \text{mod}(p_2) \setminus \text{mod}(p_1)$.
Preferring "more relevant" repairs

**Theorem**

Let $DB = \langle D \sqcup \{p_1, p_2\}, IC \rangle$ be a database, $CS = \langle C, S, R \rangle$ a context setting and $DS = \langle d, f \rangle$ a CS-sensitive distance setting in which $f$ is hereditary. If $DB_1 = \langle D \sqcup \{p_1\}, IC \rangle$ is a consistent database, $R(p_1, S) > R(p_2, S)$, and $IC \cup \{p_1, p_2\}$ is (classically) inconsistent, then no DS-repair of $DB$ contains $p_2$. 
A Natural Form of Context-Sensitive Distances

\[ CS(\mathcal{L}) = \langle C, S, R \rangle \] - a context setting

\[ g \] - an aggregation function.

\[
d_g^{CS}(I, I') = g(\{ R(p, S) \cdot |I(p) - I'(p)| \mid p \in \text{Atoms}(\mathcal{L}) \})
\]
A stronger result

Let $CS = \langle C, S, R \rangle$ be a context setting and let $DS = \langle d_g^{CS}, f \rangle$ be a distance setting, where $g$ and $f$ are hereditary aggregation functions. Let $DB = \langle D \sqcup \{p_1, p_2\}, IC \rangle$ be a database such that:

- $p_1$ is more relevant than $p_2$: $R(p_1, S) > R(p_2, S)$
- $IC \cup \{p_1, p_2\}$ is not consistent but $DB_1 = \langle D \sqcup \{p_1\}, IC \rangle$ is consistent$^1$.

Then $\Delta_{DS}(DB) = \{l_1\}$, where $l_1$ is the (unique) model of $DB_1$.

---

$^1DB_2 = \langle D \sqcup \{p_2\}, IC \rangle$ may be consistent as well, but this is not a prerequisite.
Example

<table>
<thead>
<tr>
<th>Name</th>
<th>Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bob</td>
<td>Herminengasse 13, Vienna</td>
</tr>
<tr>
<td>Bob</td>
<td>Baker Street 20, London</td>
</tr>
<tr>
<td>Alice</td>
<td>42th Street, NY</td>
</tr>
</tbody>
</table>

Integrity constraint: Name → Address

$T_{\text{UK}}^1$ - Bob lives in UK
$T_{\text{AU}}^1$ - Bob lives in AU
$T_{\text{US}}^2$ - Alice lives in US

Context: $C = \{\text{country}\}$, Range(country) = \{US, UK, AT\}

Relevance ranking for $i \in \{1, 2\}$:

$$R(T_c^i, S) = \begin{cases} 1, & \text{if } c = S(\text{country}), \\ 0.5, & \text{otherwise.} \end{cases}$$

$$R(\neg T_c^i, S) = \begin{cases} 0.5, & \text{if } c = S(\text{country}), \\ 1, & \text{otherwise.} \end{cases}$$
Example - cont.

Distance setting: \(DS = \langle d^C_{\Sigma}, \Sigma \rangle\)
State: \(S(\text{country}) = \text{UK}\)

\(T^1_{\text{UK}}\) - Bob lives in UK
\(T^1_{\text{AU}}\) - Bob lives in AU
\(T^2_{\text{US}}\) - Alice lives in US

<table>
<thead>
<tr>
<th>(I)</th>
<th>(d(I, T^1_{\text{UK}}, S))</th>
<th>(d(I, T^1_{\text{AT}}, S))</th>
<th>(d(I, \neg T^1_{\text{US}}, S))</th>
<th>(d(I, \neg T^2_{\text{UK}}, S))</th>
<th>(d(I, \neg T^2_{\text{AT}}, S))</th>
<th>(d(I, T^2_{\text{US}}, S))</th>
<th>(\delta_{DS}(I, \Gamma, S))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\emptyset)</td>
<td>1</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>({T^1_{\text{UK}}})</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>({T^1_{\text{AT}}})</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>({T^1_{\text{US}}})</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>3</td>
</tr>
<tr>
<td>(\ldots)</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>({T^1_{\text{UK}}, T^2_{\text{US}}})</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(0.5)</td>
</tr>
<tr>
<td>({T^1_{\text{AT}}, T^2_{\text{US}}})</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>(\ldots)</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
</tbody>
</table>
Future Research

- More sophisticated distances (Hausdorff, Eiter and Mannila, matching-based,...).
- Ranking functions for complex formulas:
  \[ R(\neg\psi, S) = 1 - R(\psi, S), \]
  \[ R(\psi_1 \land \psi_2, S) = \min(R(\psi_1, S), R(\psi_2, S)), \]
  \[ R(\psi_1 \lor \psi_2, S) = \max(R(\psi_1, S), R(\psi_2, S)), \]
  \[ R(\psi_1 \supset \psi_2, S) = \max(R(\neg\psi_1, S), R(\psi_2, S)). \]
- Various formalisms for modeling context.
- Implementation of a suite of tools.
An interesting connection

- Coherence theory developed by Paul Thagard in cognitive science: a computational model of cognitive ways of making sense of puzzling facts by fitting them into a "coherent" pattern.
- Formal specification of the coherence problem:
  - $F$ - set of statements
  - $L^+, L^- \in F \times F$ - sets of positive and negative constraints (e.g., $(f_1, f_2) \in L^-$ if $f_1$ is inconsistent with $f_2$).
  - $W$ - a set of weights on these constraints.
- Problem: partition $F$ into disjoint sets of accepted and rejected elements to **maximize** the weight of satisfied constraints.
Summary: Advantages of Distance-based Framework

- Faithful to CL
- Pragmatic
- Implementable
- **Portable:** can be naturally extended to other (non-classical) formalisms.
- **Flexible:** context-aware inconsistency management policies can be incorporated, as shown by our case study in database repair.
Summary: Advantages of Distance-based Framework

- Faithful to CL
- Pragmatic
- Implementable
- **Portable:** can be naturally extended to other (non-classical) formalisms.
- **Flexible:** context-aware inconsistency management policies can be incorporated, as shown by our case study in database repair.

THANK YOU!